Modeling the seasonality of mopane worm outbreaks in Limpopo, South Africa.

Lestinah Mthombeni **Mathematics and Applied Mathematics, University of Cape Town**

Motivation

For decades, rural communities in Africa have relied on mopane worm and mopane woodlands for economic livelihood. Mopane forests occur in hot, dry, and low-lying areas in the far northern parts of southern Africa. Where it occurs, the mopane tree is often the dominant tree species, often forming homogeneous stands. The mopane worm (Gonimbrasia belina) primarily, but not exclusively, feeds on mopane tree leaves, and is a source of protein and income for some people in the Limpopo province of South Africa. The outbreaks of mopane worms are strongly seasonal, emerging mostly in the rainy season.

We therefore in these work analyse the outbreaks of mopane worm by considering the seosonality. A life cycle model for worm/tree interaction based on the unification of the traditional Predator-prey model are presented(see "population dynamics model").

The schematic representations of the mopane worm life cyc

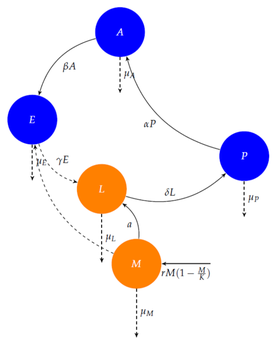

Figure 1: Flow diagram representing the model on the mopane life cycle

- The diagram consists of five compartments of which the first four are the eggs (E), larval (L), pupal (P), and a moth (A), and additional mopane leaves (M) compartment(the fifth) that larvae depend on for growth biomass.
- The nodes represents the number of each life cycle populations in each compartment. Arrows represents movements of each class or populations in the given compartment within the time t.

Seasonal varying patterns on mopane worm life cycle

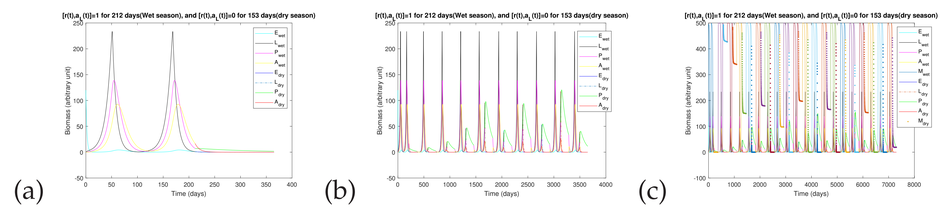

Figure 2: Simulation for the two outbreaks(wet & dry season). Pattern $(a)\&(b)$ indicate year 1 and 10 years of interactions; and (c) 20 years. Initial conditions are zeros for larva, pupa, adult, and 120 for eggs,and leaves is 100.

Summary

- The subscripts i.e M_{wet}, and M_{dry}, on graph's legend of (a), (b), and (c) represent seasons of biomasses in wet and dry season respectively. From Figure $1(a\&b)$ we see that in wet season the biomasses of egg, pupa, larva, adult each peak to the same level throughout the 10 years. It is observed, that in year 3 the mass is conserve in the system during the wet season. In dry season, the pupa biomass is highest compared to the others, with the other biomasses nearly zero.
- Graph (c), shows the beginning of dry season mopane leaves to have shed while pupa was slight active during the time. In year two from second year of dry season when pupa increase gradually, the mopane leaves could not fully sheds. From the behaviour of these interaction, we can think of too much rain which might have took place place in the previous season.

Population dynamics model

The following system of differential equations describe life cycle of mopane worm, and thus follows from Figure 1 as constructed:

$$\begin{aligned}
\frac{dE}{dt} &= \beta A - (\gamma + \mu_E)E, \\
\frac{dL}{dt} &= \gamma E + ca_M L\left(\frac{M}{M + M_{1/2}}\right) - (\delta + \mu_L)L, \\
\frac{dP}{dt} &= \delta L - (\alpha + \mu_P)P, \\
\frac{dA}{dt} &= \alpha P - (\beta + \mu_A)A, \\
\frac{dM}{dt} &= s(t)rM\left(1 - \frac{M}{K}\right) - \mu_M M - a_M \frac{ML}{M + M_{1/2}}
\end{aligned}$$

where

β	rate eggs laid
μ_E	egg mortality
γ	the rate of hatching into larvae.
μ_L	larval death rate,
$M_{1/2}$	larval half saturation constant
δ	larva to pupa transformation rate
μ_P	is mortality rate of pupae
α	pupa to adult transformation rate
μ_A	adult moth mortality rate
μ_M	mopane leave biomass inherent loss rate
K	carrying capacity of mopane leaves
r	growth rate of mopane leaves.
a_M	basic intake rate of leaves by worms

Two season(dry and wet) in a year are identified, and realising that in the dry season there are no mopane leaves, as the mopane tree sheds its leaves between May and September, we can define a piecewise function

$$s(t) = \begin{cases} 1, & 0 < d \leq 212 \text{ (October to April).} \\ 0, & 212 < d \leq 365 \text{ (May to September).} \end{cases} \quad (1)$$

where d is number of days in a year.

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